Mrs. Kim

Unit 4 Test Review

1) $\log_4 64 = \boxed{3}$

Think, $4^{\square} = 64$

2) $\log\left(\frac{1}{\sqrt{10}}\right) = \boxed{-\frac{1}{2}}$

Think, $10^{\square} = 10^{-\frac{1}{2}}$

3) $\log_2\left(\frac{1}{64}\right) = \boxed{-6}$

Think, $2^{\square} = 64^{-1}$

$2^{\square} = 2^{6(-1)}$

$2^{\square} = 2^{-6}$

4) $\log_4 192 - \log_4 3$

$= \log_4 \frac{192}{3}$

$= \log_4 \frac{\cancel{192}\ 64}{\cancel{3}\ 1}$

$= \log_4 64$

$= \boxed{3}$

Think

$\log_4 64 =$

$4^{\square} = 64$

5) $\log_{\frac{1}{3}} 27 = \boxed{-3}$

Think. $\left(\frac{1}{3}\right)^{\square} = 27$

$3^{-1\square} = 3^3$

$3^{-1\boxed{3}} = 3^3$

6) $\log (10{,}000) = \boxed{4}$

Think, $10^{\square} = 10{,}000$

7) $\log_5 \left(\frac{2\sqrt{y}\, x^3}{z^4}\right)$

$= \log_5 2\sqrt{y}\, x^3 - \log_5 z^4$

$= \log_5 2 (y)^{\frac{1}{2}} x^3 - 4 \log_5 z$

$= \log_5 2 + \log_5 y^{\frac{1}{2}} + \log_5 x^3 - 4 \log_5 z$

$= \boxed{\log_5 2 + \frac{1}{2}\log_5 y + 3\log_5 x - 4\log_5 z}$

8) $\ln \frac{ex^3}{(x-4)^2} = \ln ex^3 - \ln (x-4)^2$

$= \ln e + \ln x^3 - 2\ln(x-4)$

$= \ln e + 3\ln x - 2\ln(x-4)$

$\log_e e = 1$

$\boxed{1 + 3\ln x - 2\ln(x-4)}$

9) $3(\log x - \log y) + 4\log x$

$3\left(\log \frac{x}{y}\right) + \log x^4$

$= \log\left(\frac{x}{y}\right)^3 + \log x^4$

$= \log \frac{x^3}{y^3} + \log x^4$

$= \log \frac{x^3}{y^3} \cdot x^4$

$= \boxed{\log \frac{x^7}{y^3}}$

10) $\frac{1}{2}\ln y + 3\ln x - 3[\ln(x+2) + \ln 2]$

$\sqrt{y} \rightarrow$

$= \ln y^{\frac{1}{2}} + \ln x^3 - 3[\ln(x+2) + \ln 2]$

$= \ln(\sqrt{y})(x^3) - 3\ln(x+2) - 3\ln 2$

$= \ln(\sqrt{y})(x^3) + \ln(x+2)^{-3} + \ln 2^{-3}$

$= \ln(\sqrt{y})(x^3)(x+2)^{-3}(2^{-3})$

$= \ln \frac{x^3\sqrt{y}}{2^3(x+2)^3}$

$= \boxed{\ln \frac{x^3\sqrt{y}}{8(x+2)^3}}$

11) $25^{x-3} = 125^{2x+1}$

$5^{2(x-3)} = 5^{3(2x+1)}$

$2(x-3) = 3(2x+1)$

$2x - 6 = 6x + 3$

$-9 = 4x$

$\boxed{-\frac{9}{4} = x}$

12) $7^{x+3} = \left(\frac{1}{49}\right)^{2x+1}$

$7^{x+3} = 7^{-2(2x+1)}$

$x+3 = -2(2x+1)$

$x+3 = -4x-2$

$5x = -5$

$\boxed{x = -1}$

13) $7 - 2e^x = 5$

$-7 \qquad -7$

$\frac{-2e^x}{-2} = \frac{-2}{-2}$

$e^x = 1$

$\ln e^x = \ln 1$

$\boxed{x = 0}$

14) $\frac{3(5^{x-1})}{3} = \frac{21}{3}$

$5^{x-1} = 7$

$\log_5 5^{x-1} = \log_5 7$

$x - 1 = \log_5 7$

$x - 1 = \frac{\log 7}{\log 5}$

$x - 1 \approx \frac{0.845}{0.6989}$

$x - 1 \approx 1.209$

$+1 \qquad +1$

$\boxed{x \approx 2.209}$

15) $\frac{5\log(x-2)}{5} = \frac{11}{5}$

$\log(x-2) = \frac{11}{5}$

$10^{\log(x-2)} = 10^{\frac{11}{5}}$

$x-2 = 10^{\frac{11}{5}}$

$\boxed{x = 10^{\frac{11}{5}} + 2}$

or

$\boxed{x \approx 160.489}$

16) $\frac{3\ln(x-2)}{3} = \frac{10}{3}$

$\ln(x-2) = \frac{10}{3}$

$e^{\ln(x-2)} = e^{\frac{10}{3}}$

$x-2 = e^{\frac{10}{3}}$

$\boxed{x = e^{\frac{10}{3}} + 2}$

or

$\boxed{x \approx 30.032}$

17) $\ln 5 = \ln(x-1) - \ln(x+1)$

$\ln 5 = \ln \frac{(x-1)}{(x+1)}$

$e^{\ln 5} = e^{\ln \frac{(x-1)}{(x+1)}}$

$5 = \frac{x-1}{x+1}$

$5(x+1) = (x-1)$

$5x+5 = x-1$

$-x \quad -x$

$4x+5 = -1$

$-5 \quad -5$

$\frac{4x}{4} = \frac{-6}{4}$

$x = -\frac{3}{2}$

$\ln 5 = \ln(-\frac{3}{2}-1) - \ln(-\frac{3}{2}+1)$

$\ln 5 = \ln(-2\frac{1}{2}) - \ln(-\frac{1}{2})$

↓ Can't have − as base

$-\frac{3}{2}$ is extraneous

No Solution

18) $\ln 5 + \ln(x+3) = 1$

$\ln 5(x+3) = 1$

$e^{\ln 5(x+3)} = e^1$

$5(x+3) = e$

$5x + 15 = e$

$5x = e - 15$

$x = \frac{e-15}{5}$ (or)

$x \approx -2.456$

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19) $\log_2 5 - 1 = \log_2 (x+1)$

$-\log_2(x+1) \quad +1 \qquad -\log_2(x+1) \quad +1$

$\log_2 5 - \log_2 (x+1) = 1$

$\log_2 \frac{5}{x+1} = 1$

$2^{\log_2 \frac{5}{x+1}} = 2^1$

$\frac{5}{x+1} = 2$

$\frac{5}{x+1} \times \frac{2}{1}$

$2(x+1) = 5$

$2x + 2 = 5$

$2x = 3$

$\boxed{x = \frac{3}{2}}$

20) $\log_3 (x+2) + \log_3 x = 1$

$\log_3 (x+2)x = 1$

$\log_3 x^2+2x = 1$

$3^{\log_3 x^2+2x} = 3^1$

$x^2+2x = 3$

$x^2+2x-3 = 0$

$(x-1)(x+3) = 0$

$x=1$ | $x=-3$ extraneous

↓

can't have log of a negative #

	x	$+3$
x	x^2	$3x$
-1	$-x$	-3

Graph. Write the equation of the asymptote and give the domain and range in interval notation.

21. $f(x)=3\left(\frac{1}{2}\right)^{x-2}$

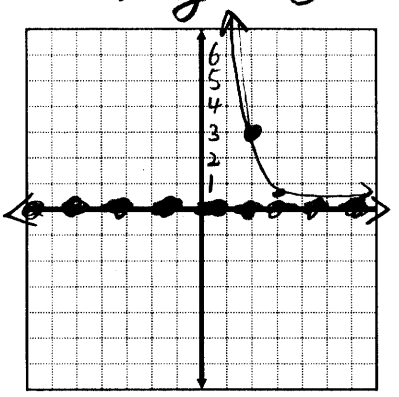
$y=3\left(\frac{1}{2}\right)^{x-2}$ → can't be 0

$y \neq 3(0)$

$y \neq 0$

asymptote → Asy: $y=0$

x	y
2	3
0	12
3	1.5



Domain: $(-\infty,\infty)$ Range: $(0,\infty)$

22. $f(x)=\ln(x-4)$

$y=\ln(x-4)$

$y=\log_e(x-4)$

$e^y=x-4$

can't be 0

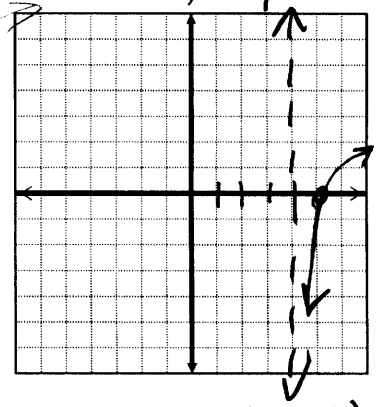
$0 \neq x-4$

$4 \neq x$

asymptote → Asy: $x=4$

x	y
5	0
11.389	2

$e^2 \approx 7.389$



Domain: $(4,\infty)$ Range: $(-\infty,\infty)$

23. $f(x)=\log_2(x+2)-1$

$y=\log_2(x+2)-1$

$y+1=\log_2(x+2)$

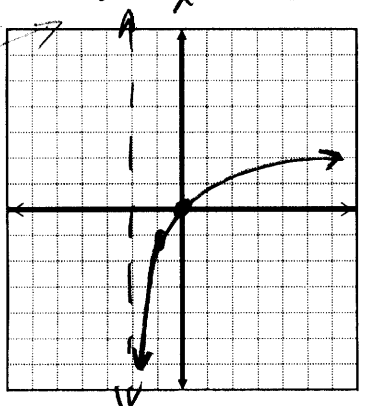
$2^{y+1}=x+2$

$0 \neq x+2$

$-2 \neq x$

asymptote → Asy: $x=-2$

x	y
-1	-1
0	0



Domain: $(-2,\infty)$ Range: $(-\infty,\infty)$

24. $f(x)=e^{x-3}+1$

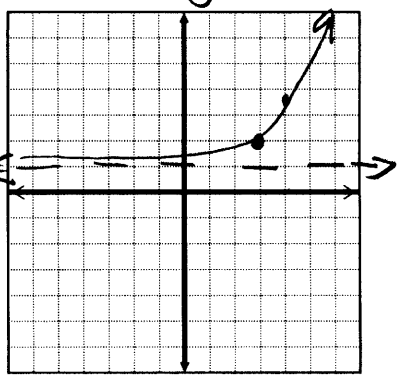
$y=e^{x-3}+1$

$y \neq 0+1$

$y \neq 1$

asymptote → Asy: $y=1$

x	y
3	2
4	3.718



Domain: $(-\infty,\infty)$ Range: $(1,\infty)$

Solve.

25. Find the amount of an investment of $5,000 at 4% compounded monthly for 8 years.

$(5,000)\left(1+\frac{0.04}{12}\right)^{12\cdot 8}$

$\approx (5,000)(1.00333333)^{96}$

≈ 6881.98

$6881.98

26. Find the amount of an investment of $10,000 at 6% compounded continuously for 5 years.

Pe^{rt}

$(10,000)(e)^{(0.06)(5)}$

$\approx 10,000\,(1.3498588) \approx$ $13,498.58